3-D Structural Design of Magnetic Actuator Using Hybrid Analysis-Based Design Optimization Method

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This paper presents an efficient design method for optimizing a 3-D structure of a magnetic actuator. To reduce a computational cost, a simple magnetic circuit is used to predict a driving performance of the magnetic actuator and to define the boundary condition for performing a detailed shape design. The finite element (FE) is created only in the design domain to prevent a rapid increase in the total number of design variables. The circuit and FE domain can be coupled by using a simple relationship between the magnetic vector potential and the magnetic flux. Two types of optimization problem are formulated to determine the circuit parameters for enhanced actuator performance and obtain a detailed structure, which is equivalent to the optimal circuit variables. Level set function is employed to obtain the optimal structure of the magnetic actuator in the FE domain and its design sensitivity is calculated using both the optimal circuit parameters and the FE-based field variables.

***Index Terms*—3-D structural design, finite element, hybrid analysis method, level set method, magnetic actuator, magnetic equivalent circuit**

I. INTRODUCTION

TOPOLOGY OPTIMIZATION method that was developed for obtaining an innovative structural design without requiring input of initial geometrical data has been successfully applied to design problem of a magnetic actuator [1]-[2]. Especially, the level set-based topology optimization method, which has advantage of the clear boundary expression, is often the focus of recent studies because even small change of the material boundary can affect the actuator performances [3]-[4]. However, such design method needs the finite element (FE) domain to predict the magnetic performance and to calculate the design sensitivity and, unfortunately, a huge computational is needed to deal with a 3-D structural design problem.

This paper presents an efficient 3-D structural design method for the magnetic actuator based on the concept of a hybrid analysis method. The magnetic equivalent circuit (MEC) and FEs are employed to perform the magneto-static analysis and determine the design sensitivity in a short period of computational time. To combine two different analysis domain, the coupling boundary condition is defined by using the relationship between the magnetic vector potential in FE domain and the circuit flux. The proposed method consists of two optimization problems to determine the optimal circuit parameters for satisfying the target performance of the magnetic actuator and to obtain the detailed structural shape of the actuator in 3-D FE-based design domain with respect to the optimized value of the circuit variables.

II. PROBLEM FORMULATION

A. Hybrid Analysis Method in 3-D Magnetic Problem

Hybrid analysis method is the circuit-field coupled method for achieving both the efficiency and the accuracy. Since the MEC and the FE analysis domain use different types of system variable, such as the circuit flux (B_{MEC}) and the magnetic vector potential ($\mathbf{A}_{\text{FE}}$), a specific coupling boundary condition is needed to combine them. In FE domain for 3-D magnetic analysis, the magnetic flux ($\mathbf{B}_{\text{FE}}$) and the magnetic vector potential has the following relationship.

$$\mathbf{B}_{\text{FE}} = B_{\text{FE},x}\mathbf{i} + B_{\text{FE},y}\mathbf{j} + B_{\text{FE},z}\mathbf{k} = \nabla \times \mathbf{A}_{\text{FE}} \tag{1}$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the normal vector for *x*, *y*, and *z* axis in the 3-D analysis domain, respectively. The magnetic potential component ($A_{\text{FE},x}, A_{\text{FE},y}, A_{\text{FE},z}$) can be defined by the combination of the magnetic flux component in three directions ($B_{\text{FE},x}, B_{\text{FE},y}, B_{\text{FE},z}$), as follows:

$$\begin{cases} A_{\text{FE},x} = a_1 B_{\text{FE},y} z + a_2 B_{\text{FE},z} y + C_1 \\ A_{\text{FE},y} = a_3 B_{\text{FE},z} x + a_4 B_{\text{FE},x} z + C_2 \\ A_{\text{FE},z} = a_5 B_{\text{FE},x} y + a_6 B_{\text{FE},y} x + C_3 \end{cases} \tag{2}$$

Since the magnetic flux is defined by the relative value of the vector potential, the constants of integration ($C_1 \sim C_3$) can be chosen to arbitrary values and the proportional coefficients ($a_1 \sim a_6$) are determined by using the following conditions.

$$a_5 - a_4 = a_1 - a_6 = a_3 - a_2 = 1 \tag{3}$$

At the coupling surfaces (S_c), the magnetic flux from the MEC and the FE domain should be same, as shown in Fig. 1, and, thus, the magnetic vector potential for performing the hybrid analysis is defined by the circuit flux.

$$\mathbf{A}_{\text{FE}}\big|_{S_c} = \mathbf{A}_{\text{FE}}\left(B_{\text{FE},i}\right) = \mathbf{A}_{\text{FE}}\left(B_{\text{MEC},i}\right) \quad \text{where} \quad i = x, y, z \tag{4}$$

Then, the circuit-field coupled analysis for predicting the magnetic performance of the actuator can be performed by the following system equations.

$$\mathbf{M}_{\text{MEC}} = \mathbf{R}_{\text{MEC}} \cdot \mathbf{B}_{\text{MEC}} \tag{5}$$

$$\nabla \times \left[\mu_0^{-1} \mu_r^{-1} \nabla \times \mathbf{A}_{\text{FE}}\left(\mathbf{B}_{\text{MEC}}\right)\right] = 0 \tag{6}$$

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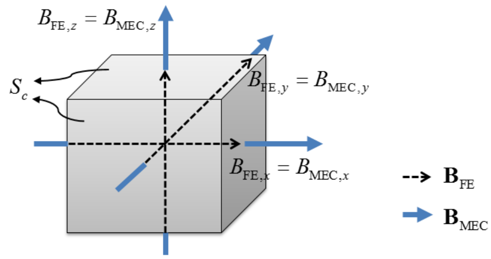


Fig. 1. Hybrid analysis domain in 3-D magnetic problem

where $\mathbf{M}_{MEC}$ and $\mathbf{R}_{MEC}$ are the magneto-motive force vector and the reluctance matrix of the MEC.

B. Optimization Problem Formulation

The circuit optimization problem is formulated to maximize the driving performance of the actuator (f_{MEC}), as follows:

$$\begin{aligned} &\underset{\mathbf{R}_{MEC}}{\text{maximize}} \ f_{MEC}\left(\mathbf{R}_{MEC}, \mathbf{B}_{MEC}\right) \\ &\text{subject to } \mathbf{R}_{lower} \le \mathbf{R}_{MEC} \le \mathbf{R}_{upper} \end{aligned} \tag{7}$$

where $\mathbf{R}_{lower}$ and $\mathbf{R}_{upper}$ are the lower and the upper limit of the circuit reluctance, which is defined by geometric constraints of the actuator. After determining the optimal circuit flux ($\mathbf{B}_{MEC,opt}$) and the coupling boundary condition for FE domain ($\mathbf{A}_{FE}\left(\mathbf{B}_{MEC,opt}\right)$), an another optimization problem is solved for obtaining the detailed structural design in the design domain by the following formulation.

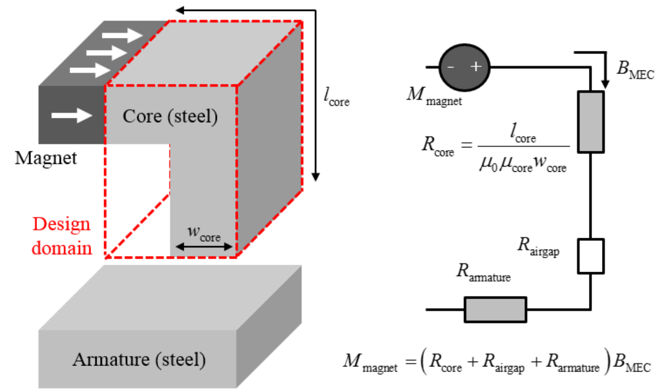
$$\begin{aligned} &\underset{\phi_{FE}}{\text{minimize}} \left|\mathbf{B}_{MEC,opt} - \mathbf{B}_{FE}\left(\phi_{FE}\right)\right| \\ &\text{subject to } \mathrm{VF}\left(\phi_{FE}\right) - \mathrm{VF}_0 \le 0 \end{aligned} \tag{8}$$

where ϕ_{FE} is the level set function, which is employed as a topological design variable [4], and VF is the volume fraction constraint of the magnetic material. The detailed actuator shape for providing the same flux as the optimal circuit flux can be obtained through the optimization.

III. Design Example

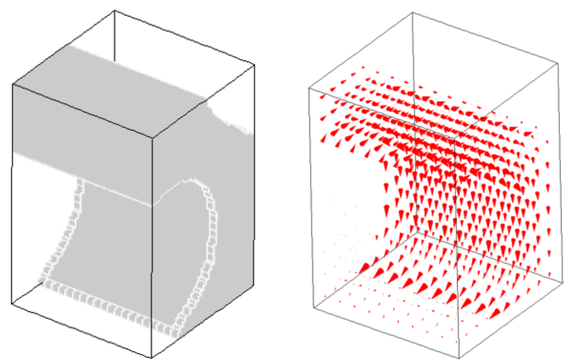
The proposed design method was applied to the simple actuator design problem [5] whose initial design and the design domain are illustrated in Fig. 2 (a). The optimization was performed to maximize the magnetic force of the armature and the MEC for the circuit optimization is shown in Fig. 2 (b). To maximize the total amount of the magnetic flux, the core reluctance (R_{core}) should be minimized and, thus, it is obvious that the core width (w_{core}) and the core length (l_{core}) reached to their maximum and minimum values through the circuit optimization.

If the design optimization was performed in the full FE analysis domain, at least 24,000 FE have needed to represent the PM, the armature, and the free space. In the proposed method, total number of FE in the design domain is only 8,000. Fig. 3 depicts the optimal shape design of the core and the inner magnetic flux path. To prevent the flux concentration at the edge of the core and provide a sufficient flux to the



(a) (b)

Fig. 2. C-core actuator: (a) initial design and design domain (b) MEC



(a) (b)

Fig. 3. Optimal core design obtained by the proposed method: (a) optimal shape (b) magnetic flux path

armature, the outer boundary of the core became a smooth curve and the amount of the steel is located near the armature. This optimal design could increase the magnetic force by 23.1%, compared with the initial design.

IV. Conclusion

A new efficient optimization method to obtain the 3-D structural design of a magnetic actuator is presented. Performing the simple design example, it was confirmed that the proposed method could reduce more than 65% of the design variable and optimization time while providing the reasonable optimal shape of the actuator.

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